On an extension of General Coordinate Transformations Algebra.

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Abstract

An extension of the General Coordinate Transformations algebra is constructed by means geometrical consistency conditions. An class of infinite invariants is derived. In particular we construct the consistent extension of the gravitational anomaly for each even dimension. The new contributions for these anomalies allow to define an improved Ward operator for which the symmetry is restored.

1 Introduction

Over the course of the last century it has become clear that both elementary particle physics and relativity theories are based on the notion of symmetries. These symmetries become manifest in that the "laws of nature" are invariant under spacetime transformations and/or gauge transformations. In fact the dynamics of physical theories is deeply intertwined with space-time reparametrization invariance; indeed it assures the independence of the physics from the frames, and so endorses the universality requirements. These fulfillments can be assured in several ways, the most canonical ones are performed or via finite reparametrizations (change of charts) or through infinitesimal transformations, which allow the use of the General Coordinate Transformations (diffeomorphism) algebra within the most common treatments of the physical theories (see Ref[1] for an introduction)

Moreover one can find in the Literature a widespread of intermediate approaches; we recall the use of discrete coordinate transformation [2],[3] , but we focus the attention to the algebra extension methods, which, in the past Literature found a great popularity within the holomorphic two dimensional models with the discovery of the so-called $\mathcal{W}$-algebras [4] [5]. This approach appears, after many years of investigations, as a clever improvement of the infinitesimal transformations approach. So their extension to higher dimensions might be interesting in those cases where the smoothness of the coordinates change is not verified. We shall perform the extensions via the B.R.S. [6] consistency requirements, inserting first the infinitesimal contributions, and then adding new further terms, which define the extension, whose algebraic properties will be defined by the nilpotency of the algebra transformations. It is easy to realize that this algebra process is not a simple task, and a priori depends on the space-time geometries, so that the roads covered by a well defined deformation are not generally straight, but their tortuosity have to be settled by gates defined be precise algebraic supplementary conditions. So we can expect that this treatment will have repercussions on the physical landscape. This can be manifested by the Lagrangians and Anomalies changes. It would be interesting if this new expansion would produce new welcome physics perspectives.

In Section (2) we shall perform an infinite expansion, whose consistency constraints greatly reduce the B.R.S. ghosts from infinity to two, with well defined geometrical properties.

Then in Section (3) we calculate the anomalies change. This improvement reveals a welcome intertwining property among the ghosts, which, at any dimension, permits to define a combined, anomaly free, Ward operator.

Few conclusions are reported in Section (4), and in the Appendix (A) we report the calculations which lead to the explicit expressions of the anomalies at any dimensions quoted in Section (3)

2 The diffeomorphism algebra extension in a B.R.S. approach

Our treatment begins supposing that the Riemmanian connection $\Gamma^{\nu}_{(\mu,\lambda)}(x)$, due to an improvement, satisfies the infinitesimal (B.R.S) extended reparametrization transformation:

$$\mathcal{S}\Gamma^{\nu}_{(\mu,\lambda)}(x) = \mathcal{C}^{\sigma}(x)\partial_{\sigma}\Gamma^{\nu}_{(\mu,\lambda)}(x) + \partial_{\mu}\mathcal{C}^{\sigma}\Gamma^{\nu}_{(\sigma,\lambda)}(x) + \partial_{\lambda}\mathcal{C}^{\sigma}(x)\Gamma^{\nu}_{(\mu,\sigma)}(x) - \partial_{\sigma}\mathcal{C}^{\nu}(x)\Gamma^{\sigma}_{(\mu,\lambda)}(x) - \partial_{\mu}\partial_{\lambda}\mathcal{C}^{\sigma}(x) + \Omega^{\nu}_{(\mu,\lambda)}(x) \tag{2.1}$$

where $\Omega^{\nu}_{(\mu,\lambda)}(x)$ is a new (general) tensorial extension term (absent in the ordinary general coordinate transformations algebra), which could reproduce higher diffeomorphisms corrections, and $\mathcal{C}^{\mu}(x)$ is the usual diffeomorphism B.R.S. ghost.

Equation (2.1) has a full covariant rewriting, in terms of the Riemmaniann curvature $\mathcal{R}^{(\nu)}_{([\rho,\lambda],\mu)}(x)$, as:

$$\mathcal{S}\Gamma^{\nu}_{(\mu,\lambda)}(x) = -\mathcal{D}_{\lambda}\mathcal{D}_{\mu}\mathcal{C}^{\nu}(x) + \mathcal{C}^{(\rho)}(x)\mathcal{R}^{(\nu)}_{([\rho,\lambda],\mu)}(x) + \Omega^{\nu}_{(\lambda,\mu)}(x) \tag{2.2}$$

where $\mathcal{D}_{\lambda}$ indicates the the covariant derivative.

In this spirit the ghost $\mathcal{C}^{\mu}(x)$ must undergo a new extended transformation:

$$\mathcal{S}\mathcal{C}^{\mu}(x) = \mathcal{C}^{\lambda}(x)\mathcal{D}_{\lambda}\mathcal{C}^{\mu}(x) + \mathcal{X}^{\mu}(x) \tag{2.3}$$

So,we have a new law for the B.R.S curvature transformation:

$$\begin{aligned}
\mathcal{S}\mathcal{R}^{\sigma}_{([\nu,\mu],\rho)}(x) &= \mathcal{D}_{\nu}\left(\mathcal{S}\Gamma^{\sigma}_{(\mu,\rho)}(x)\right) - \mathcal{D}_{\mu}\left(\mathcal{S}\Gamma^{\sigma}_{(\nu,\rho)}(x)\right)\\
&= \mathcal{R}^{\lambda}_{([\mu,\nu]\rho)}(x)\mathcal{D}_{\lambda}\mathcal{C}^{\sigma}(x) - \mathcal{R}^{\sigma}_{([\mu,\nu],\lambda)}(x)\mathcal{D}_{\rho}\mathcal{C}^{\lambda}(x) + \mathcal{D}_{\nu}\mathcal{C}^{\lambda}(x)\mathcal{R}^{\sigma}_{([\lambda,\mu],\rho)}(x)\\
&+ \mathcal{C}^{\lambda}(x)\mathcal{D}_{\nu}\mathcal{R}^{\sigma}_{([\lambda,\mu],\rho)}(x) - \mathcal{D}_{\mu}\mathcal{C}^{\lambda}\mathcal{R}^{\sigma}_{([\lambda,\nu]\rho)}(x) - \mathcal{C}^{\lambda}(x)\mathcal{D}_{\mu}\mathcal{R}^{\sigma}_{([\lambda,\nu]\rho)}(x)\\
&+ \mathcal{D}_{\nu}\Omega^{\sigma}_{(\mu,\rho)}(x) - \mathcal{D}_{\mu}\Omega^{\sigma}_{(\nu,\rho)}(x)
\end{aligned} \tag{2.4}$$

In the B.R.S. formalism the consistency is fulfilled by the algebra nilpotency requirements, which give for the connection extensions the B.R.S. full covariant transformations:

$$\begin{aligned}
&\mathcal{S}\Omega^{\nu}_{(\mu,\lambda)}(x) = \mathcal{C}^{\sigma}(x)\mathcal{D}_{\sigma}\Omega^{\nu}_{(\mu,\lambda)}(x) + \mathcal{D}_{\mu}\mathcal{C}^{\sigma}\Omega^{\nu}_{(\sigma,\lambda)}(x)\\
&+\mathcal{D}_{\lambda}\mathcal{C}^{\sigma}(x)\Omega^{\nu}_{(\mu,\sigma)}(x) - \mathcal{D}_{\sigma}\mathcal{C}^{\nu}(x)\Omega^{\sigma}_{(\mu,\lambda)}(x) + \mathcal{D}_{\mu}\mathcal{D}_{\lambda}\mathcal{X}^{\nu}(x) - \mathcal{X}^{\sigma}(x)\mathcal{R}^{\nu}_{([\sigma,\mu],\lambda)}(x)
\end{aligned} \tag{2.5}$$

On the other hand. the ghost algebra extension $\mathcal{X}^{\sigma}(x)$ has (due to the nilpotency of Equation (2.3)) a B.R.S. transformation:

$$\mathcal{S}\mathcal{X}^{\sigma}(x) = \mathcal{C}^{\lambda}(x)\partial_{\lambda}\mathcal{X}^{\sigma}(x) - \partial_{\lambda}\mathcal{C}^{\sigma}(x)\mathcal{X}^{\lambda}(x) \tag{2.6}$$

It is trivial to verify $\mathcal{S}^2\Omega^{\nu}_{(\mu,\lambda)}(x) = 0$, $\mathcal{S}^2\mathcal{X}^{\sigma}(x) = 0$.

In this approach we suppose that the ghost expansion term $\mathcal{X}^{\mu}(x)$ can be interpreted as a consequence of the connection extension $\Omega^{(\rho)}_{(\mu,\nu)}(x)$ term, and goes to zero when this one vanishes. So we expand $\mathcal{X}^{\mu}(x)$ in terms of the connection extension and its derivatives, with coefficients higher order ghosts $\mathcal{C}^{(\lambda_1,\cdots,\lambda_j)}(x)$ with Φ,Π charge equal to one, and we choose

$$\mathcal{X}^{\sigma}(x) = \sum_{j=1}^{\infty}\left[\mathcal{C}^{(\eta_1,\cdots,\eta_j)}(x)\left(\mathcal{D}_{\eta_1}\cdots\mathcal{D}_{\eta_{j-2}}\Omega^{\sigma}_{(\eta_{j-1}\eta_j)}(x)\right)\right] \tag{2.7}$$

We remark that the $\mathcal{C}^{(\eta_1,\cdots,\eta_j)}(x)$ ghosts tensors order starts from two and goes to infinity.

At this stage it is evident that the Equation (2.7) must be consistent with Equations (2.6)and (2.5) which should provide the B.R.S. variations of $\mathcal{C}^{(\eta_1,\cdots,\eta_j)}(x)$, which can be derived picking out the terms $\left(\mathcal{D}_{\eta_1}\cdots\mathcal{D}_{\eta_{j-2}}\Omega^{\sigma}_{(\eta_{j-1}\eta_j)}(x)\right)$, (in which is placed the free σ index).

This inspection is (almost) always successful, but encounter an obstacle provided by the presence of the last term of the $\Omega^{\sigma}_{(\mu,\nu)}(x)$ B.R.S transformations $(\mathcal{X}^{(\lambda)}(x)\mathcal{R}^{(\sigma)}_{([\lambda,\rho],\eta)}(x))$,in Equation (2.5), which exhibits, the free index placed in the top of the curvature, and *not* on the $\mathcal{X}^{\lambda}(x)$ term. So, taking into account Equation (2.7)), the free index misplacement forbids the complete extraction of these terms.

Indeed must be natural suppose that this extension process is not an easy trip, and the geometry 'a priori' puts barriers to this program. So, in order to bypass this obstacle, we have to fix conditions, which would assure the success of the operation.

The most simple requirement is to fix (this is one of the key points of the paper):

$$\mathcal{X}^{(\lambda)}(x)\mathcal{R}^{(\mu)}_{([\lambda,\rho],\sigma)}(x)=0 \tag{2.8}$$

[1]

But, substituting Equation (2.7) into Equation (2.8)we get:

$$\sum_{j=1}^{\infty}\left[\mathcal{C}^{(\eta_1,\cdots,\eta_j)}(x)\left(\mathcal{D}_{\eta_1},\cdots,\mathcal{D}_{\eta_{j-2}}\Omega^{\lambda}_{(\eta_{j-1}\eta_j)}(x)\right)\right]\mathcal{R}^{(\mu)}_{([\lambda,\rho],\sigma)}(x)=0 \tag{2.10}$$

The constraint in Equation (2.10) can be verified (due to its Faddeev-Popov content) if we impose:

$$\begin{aligned}
&\left[\left(\mathcal{D}_{\eta_1}\cdots\mathcal{D}_{\eta_{j-2}}\Omega^{\lambda}_{(\eta_{j-1}\eta_j)}(x)\right)\right]\mathcal{R}^{(\mu)}_{([\lambda,\rho],\sigma)}(x)=\mathcal{M}^{(\mu)}_{((\eta_1,\cdots,\eta_j),(\tau_1,\cdots,\tau_j);\rho,\lambda)}(x)\mathcal{C}^{(\eta_1,\cdots,\eta_j)}(x)\\
&\mathcal{M}^{(\mu)}_{((\eta_1,\cdots,\eta_j),(\tau_1,\cdots,\tau_j);\rho,\lambda)}(x)=\mathcal{M}^{(\mu)}_{((\tau_1,\cdots,\tau_j),(\eta_1,\cdots,\eta_j);\rho,\lambda)}(x)\\
&\forall j\geq 1
\end{aligned} \tag{2.11}$$

We underline that Equation (2.11), if inverted, defines the $\mathcal{C}^{(\eta_1,\cdots,\eta_j)}(x)$ ghosts in term of the $\Omega^{\lambda}_{(\rho,\sigma)}(x)$ covariant derivatives. This fact has to be examined later in all its details.

Before this we underline that,if the condition (2.8) is fulfilled, its B.R.S. transformation too must be zero; that is:

$$\mathcal{S}\left(\mathcal{X}^{(\lambda)}(x)\mathcal{R}^{(\mu)}_{([\lambda,\rho],\sigma)}(x)\right)=0 \tag{2.12}$$

so, taking into account Equation (2.8), only the terms coming from the connection algebra extension survive, so we must require:

$$\mathcal{X}^{(\lambda)}(x)\left(D_{\lambda}\Omega^{(\mu)}_{(\rho,\sigma)}(x)-\mathcal{D}_{\rho}\Omega^{(\mu)}_{(\lambda,\sigma)}(x)\right)=0 \tag{2.13}$$

A way out can be adopted requiring:

$$\Omega^{(\mu)}_{(\rho,\sigma)}(x)=\mathcal{D}_{\rho}\mathcal{K}^{(\mu)}_{(\sigma)}(x) \tag{2.14}$$

so it is easy to verify,if we assume Equations (2.8) (2.14):

$$\begin{aligned}
\mathcal{S}\left(\mathcal{X}^{(\lambda)}(x)\mathcal{R}^{(\mu)}_{([\lambda,\rho],\sigma)}(x)\right)&=\mathcal{X}^{(\lambda)}(x)\left(\left[D_{\lambda},\mathcal{D}_{\rho}\right]\mathcal{K}^{(\mu)}_{(\sigma)}(x)\right)\\
&=\mathcal{X}^{(\lambda)}(x)\left(\mathcal{R}^{(\eta)}_{([\lambda,\rho],\sigma)}(x)\mathcal{K}^{(\mu)}_{(\eta)}(x)-\mathcal{R}^{(\mu)}_{([\lambda,\rho],\eta)}(x)\mathcal{K}^{(\eta)}_{(\sigma)}(x)\right)=0
\end{aligned} \tag{2.15}$$

[1]an alternative second solution can be given choosing:

$$\mathcal{X}^{(\lambda)}(x)\mathcal{R}^{(\mu)}_{([\lambda,\rho],\sigma)}(x)=\mathcal{X}^{(\mu)}(x)\mathcal{G}_{(\rho,\sigma)}(x) \tag{2.9}$$

which can be appealing in case of Einstein spaces (where we have neglect the terms coming from the non extended diffeomorphism algebra, which due to Equation (2.8) are zero)

Now from Equations (2.5) and (2.14) we can derive the $\mathcal{K}^{(\mu)}_{(\sigma)}(x)$ B.R.S. variation:

$$\mathcal{S}\mathcal{K}^{(\mu)}_{(\sigma)}(x) = \mathcal{C}^{(\lambda)}\mathcal{D}_{(\lambda)}\mathcal{K}^{(\mu)}_{(\sigma)}(x) + \mathcal{D}_{(\sigma)}\mathcal{C}^{(\lambda)}\mathcal{K}^{(\mu)}_{(\lambda)}(x) - \mathcal{D}_{(\lambda)}\mathcal{C}^{(\mu)}\mathcal{K}^{(\lambda)}_{(\lambda)}(x) + \mathcal{D}_{(\sigma)}\mathcal{X}^{(\mu)}(x) - \mathcal{K}^{(\eta)}_{(\sigma)}(x)\mathcal{K}^{(\mu)}_{(\eta)}(x) \tag{2.16}$$

So, defining $\mathcal{M}^{((\eta_1,\cdots,\eta_j),(\tau_1,\cdots,\tau_j);\rho,\lambda)}_{(\mu)}(x)$ such that:

$$\mathcal{M}^{(\mu)}_{((\eta_1,\cdots,\eta_j),(\tau_1,\cdots,\tau_j);\rho,\lambda)}(x)\mathcal{M}^{((\eta_1,\cdots,\eta_j),(\tau'_1,\cdots,\tau'_j);\rho',\lambda')}_{(\mu')}(x) = \delta^{(\tau'_1,\cdots,\tau'_j)}_{(\tau_1,\cdots,\tau_j)}\delta^{\mu}_{\mu'}\delta^{\rho'}_{\rho}\delta^{\lambda'}_{\lambda} \tag{2.17}$$

the Equation, after taking into account the Equation (2.14), the Equation(2.11) can be inverted, and we obtain the expressions of the $\mathcal{C}^{(\tau_1,\cdots,\tau_j)}(x)$ in terms of the $\mathcal{K}^{\sigma}_{\rho}(x)$ connection extension parameters and their covariant derivatives.

$$\mathcal{C}^{(\eta_1,\cdots,\eta_j)}(x) = \mathcal{M}^{((\eta_1,\cdots,\eta_j),(\tau_1,\cdots,\tau_j);\rho,\lambda)}_{(\mu)}(x)\Bigg(\mathcal{D}_{\tau_1},\cdots,\mathcal{D}_{\tau_{j-1}}\mathcal{K}^{\sigma}_{(\tau_j)}(x)\Bigg)\mathcal{R}^{(\mu)}_{([\sigma,\rho],\lambda)}(x) \tag{2.18}$$

with this we have the parameters collapse, which, from an infinite multitude $(\mathcal{C}^{(\eta_1,\cdots,\eta_j)}(x), j = 1,\cdots,\infty)$, are reduced only to a couple $(\mathcal{C}^{(\eta)}(x), \mathcal{K}^{(\rho)}(x))$.

So the breaking term $\mathcal{X}^{\sigma}(x)$ is takes its final form:

$$\mathcal{X}^{\sigma}(x) \quad = \quad \sum_{j=1}^{\infty}\Bigg[\mathcal{M}^{((\eta_1,\cdots,\eta_j),(\tau_1,\cdots,\tau_j);\rho,\eta)}_{(\mu)}(x)\Bigg(\mathcal{D}_{\tau_1},\cdots,\mathcal{D}_{\tau_{j-1}}\mathcal{K}^{\lambda}_{(\tau_j)}(x)\Bigg)\mathcal{R}^{(\mu)}_{([\eta,\rho],\lambda)}(x)\Bigg(\mathcal{D}_{\eta_1},\cdots,\mathcal{D}_{\eta_{j-1}}\mathcal{K}^{\sigma}_{(\eta_j)}(x)\Bigg)\Bigg] \tag{2.19}$$

In the next Section we shall apply this extension to a new calculation of the Gravitational Anomalies[7] . We shall see that, within our treatment, the $\mathcal{X}^{\sigma}(x)$ term will play a secondary role, so we skip any further investigation on the $\mathcal{M}^{((\eta_1,\cdots,\eta_j),(\tau_1,\cdots,\tau_j);\rho,\lambda)}_{(\mu)}(x)$ terms.

3 A B.R.S. approach on the Characteristic Classes and Gravitational Anomalies

We here rephrase in this context, an approach of the present author[8] to derive invariants in an abstract way directly from the B.R.S. diffeomorphisms algebra.Then, a trick we shall see later, will permit to link them to the world of Field Theory.

We start from the study of the covariant ghost derivative [9]:

$$\mathcal{D}_{(\mu)}\mathcal{C}^{(\nu)}(x) \equiv \partial_{(\mu)}\mathcal{C}^{(\nu)}(x) - \Gamma^{(\nu)}_{(\mu,\lambda)}(x)\mathcal{C}^{(\lambda)}(x) \tag{3.20}$$

Its B.R.S. variation takes the form:

$$\mathcal{S}\mathcal{D}_{\nu}\mathcal{C}^{\mu}(x) = \Bigg(\Lambda^{\mu}_{\nu}(x) + \mathcal{D}_{\nu}\mathcal{C}^{\lambda}(x)\mathcal{D}_{\lambda}\mathcal{C}^{\nu}(x)\Bigg) \tag{3.21}$$

where:

$$\Lambda^{(\nu)}_{(\mu)}(x) = \frac{1}{2}\mathcal{C}^{(\rho)}(x)\mathcal{C}^{(\sigma)}(x)\mathcal{R}^{(\nu)}_{([\rho,\sigma],\mu)}(x) + \mathcal{D}_{\mu}\mathcal{X}^{\mu} + \mathcal{C}^{\lambda}(x)\Omega^{\nu}_{(\lambda,\mu)}(x) \tag{3.22}$$

Its variation is astonishing simple :

$$\mathcal{S}\Lambda^{\mu}_{\nu}(x) = \mathcal{D}_{\nu}\mathcal{C}^{\lambda}(x)\Lambda^{\mu}_{\lambda}(x) - \Lambda^{\lambda}_{\nu}(x)\mathcal{D}_{\lambda}\mathcal{C}^{\mu}(x) \equiv \Bigg[\mathcal{D}\mathcal{C}(x), \Lambda(x)\Bigg]^{\mu}_{\nu} \tag{3.23}$$

The fundamental result we get from Equation (3.23) is:

$$\mathcal{S}Tr\Bigg[\Lambda^{n}(x)\Bigg] = 0 \tag{3.24}$$

for all $n \geq 1$[2] So we get an infinity of local invariant under B.R.S.diffeomorphism. It is obvious to realize that this property is verified even if the infinitesimal transformations are extended or not. In the case of usual diffeomorphisms we obtain:

$$\overset{\circ}{\Lambda}{}^{(\nu)}_{(\mu)}(x) = \frac{1}{2}\mathcal{C}^{(\rho)}(x)\mathcal{C}^{(\sigma)}(x)\mathcal{R}^{(\nu)}_{([\rho,\sigma],\mu)}(x) \tag{3.27}$$

and the invariants provide a new B.R.S. way of thinking of the Chern characteristic classes. The present strategy allows to go beyond the usual linear transformations and to explore what is next to the habitual infinitesimal border.

In our framework, the most compelling question is that these invariants are cohomology elements or not.

We shall analyze this problem, constructing the solutions for each n, as coboundary polynomials,with specific and particular properties.

First of all we state here the main result (whose detailed calculations are reported in the Appendix (A)) we shall use here:

Results 3.1 *The term* $Tr\left[\Lambda^{(n)}(x)\right]$ *is a total coboundary and take the general expression:*

$$\begin{aligned}
Tr\left[\Lambda^{(n)}(x)\right] &= \mathcal{S}Tr\widehat{\Sigma}^{(2n-1)}(x) \\
\widehat{\Sigma}^{(2n-1)}(x) &\equiv \sum_{r=0}^{(n-1)} Tr\left[\mathcal{DC}(x)\left\{\left(-\Big(\mathcal{SDC}(x)\Big)^{(n-1-r)} + \Lambda^{(n-1-r)}(x)\right)\Psi^{(r)}(x)\right)\right. \\
&+ \left.\left.\frac{1}{(2r+1)}\left(\Big(\mathcal{DC}(x)\mathcal{DC}(x)\Big)^{(r)}\Big(\mathcal{SDC}(x)\Big)^{(n-r-1)}\right)\right\}\right]
\end{aligned} \tag{3.28}$$

where:

$$\begin{aligned}
\Psi^{(0)}(x) &= 1 \\
\Psi^{(1)}(x) &= (\mathcal{DC}(x)\mathcal{DC}(x)) \\
\Psi^{(2)}(x) &= \Big(\mathcal{DC}(x)\mathcal{S}(\mathcal{DC}(x)\mathcal{DC}(x)) + (\mathcal{DC}(x)\mathcal{DC}(x))^2\Big) \\
&= \cdots \\
\Psi^{(r)}(x) &= \mathcal{DC}(x)\mathcal{S}\Psi^{(r-1)}(x) + (\mathcal{DC}(x)\mathcal{DC}(x))^{(r)} \\
r &= 1,\cdots n, \qquad \mathcal{S}\Psi^{(0)}(x) = 0 \\
& \Phi\Pi\left[\Psi^{(r)}(x)\right] = 2r
\end{aligned} \tag{3.29}$$

The calculations which lead to the previous conclusions are reported in the Appendix (A).

With these results we shall be able to construct the gravitational anomalies in all even dimensions.

we give some result for the lowest value:

$$\begin{aligned}
Tr\left[\Lambda^2(x)\right] &= \mathcal{S}Tr\left(\mathcal{DC}(x)\Lambda(x) + \frac{1}{3}\mathcal{DC}(x)\mathcal{DC}(x)\mathcal{DC}(x)\right) \\
Tr\left[\Lambda^3(x)\right] &= \mathcal{S}Tr\left(\mathcal{DC}(x)\Lambda^2(x) + \frac{1}{2}\mathcal{DC}(x)\mathcal{DC}(x)\mathcal{DC}(x)\Lambda(x) - \frac{1}{10}\mathcal{DC}(x)\mathcal{DC}(x)\mathcal{DC}(x)\mathcal{DC}(x)\mathcal{DC}(x)\right)
\end{aligned}$$

[2] For sake of brevity we introduce the notations:

$$\Big(\mathcal{DC}(x)^m\Big)^{\mu}_{\nu}(x) \equiv \Big(\mathcal{D}_\nu\mathcal{C}^{\rho_1}(x)\mathcal{D}_{\rho_1}\mathcal{C}^{\rho_2}(x)\cdots\mathcal{D}_{\rho_m}\mathcal{C}^{\mu}(x)\Big) \tag{3.25}$$

$$\Big(\Lambda(x)^n\Big)^{\mu}_{\nu}(x) \equiv \Big(\Lambda^{\sigma_1}_{\nu}(x)\Lambda^{\rho_2}_{\sigma_1}(x)\cdots\Lambda^{\mu}_{\sigma_n}(x)\Big) \tag{3.26}$$

$$Tr\left[\Lambda^4(x)\right] = \mathcal{S}Tr\Bigg(\mathcal{D}\mathcal{C}(x)\Lambda^3(x) + +\frac{1}{5}\mathcal{D}\mathcal{C}(x)\Lambda(x)\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\Lambda(x) + \frac{2}{5}\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\Lambda^2(x)$$
$$+ \frac{1}{5}\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\Lambda(x) - \frac{1}{35}\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\Bigg) \tag{3.30}$$

The previous results are general and hold for all $\mathcal{X}^\mu(x)$ and $\Omega^\sigma_{(\mu,\nu)}(x)$ which obey the previous B.R.S transformations, that is for whatever consistent extension.

We have seen in past[8], that with algebraic techniques, writing the common derivative operator in the Fock space[10], in term of B.R.S ghosts and operator:,

$$\partial_\mu = \left\{\delta_{B.R.S.}, \frac{\partial}{\partial \mathcal{C}^{(\mu)}(x)}\right\} \tag{3.31}$$

(where $\delta_{B.R.S.}$ is the B.R.S. functional operator) *it is possible to relate local B.R.S. totally invariant polynomials, to invariant objects modulo total derivatives[9], and to derive all the elements of the so called "descent Equations"[11].* We remark that the highest underived $\mathcal{C}^\mu(x)$ ghost content of the previous object is the local parallel term of the invariant $Tr\left[\left(\mathcal{C}^\mu(x)\mathcal{C}^\nu(x)\mathcal{R}^\sigma_{([\mu,\nu],\rho)}(x)\right)^n\right]$, so our trick allows to extend the idea of the so-called "Russian formula" idea in presence of extensions too, and to write in a general way Topological Actions and Anomalies of the extended symmetries in fixed dimensions. So, if $\Delta^{q,\natural}_j(x)$ are the total local invariants with Faddeev- Popov (Φ,Π) charge equal to q and form degree equal to j, then the invariants (modulo total derivatives) with Φ,Π charge p and degree n takes the general expression[8]:

$$\Delta^p_n(x) = \Delta^{p,\natural}_n(x) + \sum_{j=1}^{n}(-1)^j\frac{1}{j!}dx^1\wedge\cdots\wedge dx^j\frac{\partial^j\Delta^{p+j,\natural}_{d-j}(x)}{\partial\mathcal{C}^{\mu_1}(x)\cdots\partial\mathcal{C}^{\mu_j}(x)} + \mathbf{d}\widehat{\Delta}^p_{n-1}(x) + \mathcal{S}\widehat{\Delta}^{p-1}_n(x) \tag{3.32}$$

So the more direct application of this approach, is the direct calculations of Topological Actions and Anomalies of models whose local symmetries are the reparametrizations:

So, if we calculate the Lagrangian anomalies[7], from the previous coboundary terms, we derive, besides the usual term, a further one whose Φ,Π charge is carried by the $\Omega^\sigma_{(\mu,\nu)}(x)$ ghost term, and we have to remark that *,in any dimension, the anomaly is independent from the $\mathcal{X}^\mu(x)$ invariant contributions, since in Eq (2.7) expansion no first order underived $\mathcal{C}^\mu(x)$ ghost is present.* This fact justifies the discussion cut we have performed at the end of the Section (2), and will have relevance in the following.

For example in the two dimensional case, the anomaly takes the form:

$$\Delta^\natural_{[\mu,\nu]}(x)dx^\mu\wedge dx^\nu = \frac{1}{2}dx^\mu\wedge dx^\nu\frac{\partial}{\partial\mathcal{C}^\mu(x)}\frac{\partial}{\partial\mathcal{C}^\nu(x)}\Bigg(Tr\Bigg(\mathcal{D}\mathcal{C}(x)(\frac{1}{2}\mathcal{C}(x)\mathcal{C}(x)\mathcal{R}(x) + \mathcal{C}(x)\mathcal{D}\mathcal{K}(x) + \mathcal{D}\mathcal{X}(x))$$
$$+ \frac{1}{3}\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\Bigg)\Bigg) \tag{3.33}$$

and we derive:

$$\Delta_{[\mu,\nu]}(x)dx^\mu\wedge dx^\nu = \epsilon^{\mu,\nu}(x)\left(-\mathcal{D}_\rho\mathcal{C}^\sigma(x) + \mathcal{K}^\sigma_\rho(x)\right)\mathcal{R}^\rho_{([\mu,\nu],\sigma)}(x)dx^2 \tag{3.34}$$

where we have adopted the full covariant way adopted in the Reference [12].

So if we embed this anomaly within a B.R.S. Lagrangian framework with an improved quantum Topological Action Γ , the Quantum Action Principle[13] fix this anomaly as:

$$\delta_{B.R.S}\Gamma = \int \Delta_{[\mu,\nu]}(x)dx^\mu\wedge dx^\nu \tag{3.35}$$

so, introducing the Ward operators:

$$\mathcal{W}_\mu(x) \equiv \left\{ \frac{\partial}{\partial \mathcal{C}^\mu(x)}, \delta_{B.R.S} \right\}; \qquad \mathcal{W}^\sigma_\mu(x) \equiv \left\{ \frac{\partial}{\partial \mathcal{K}^\mu_\sigma(x)}, \delta_{B.R.S} \right\} \tag{3.36}$$

the previous calculations give:

$$\mathcal{W}_\mu(x)\Gamma = \mathcal{D}_\sigma \mathcal{W}^\sigma_\mu(x)\Gamma \tag{3.37}$$

at the anomaly level.

So it is possible to define an improved local Ward operator $\widetilde{\mathcal{W}}_\mu(x) \equiv \left(\mathcal{W}_\mu(x) - \mathcal{D}_\sigma \mathcal{W}^\sigma_\mu(x) \right)$ such that it is possible to have

$$\widetilde{\mathcal{W}}_\mu(x)\Gamma = 0 \tag{3.38}$$

for which a symmetry is locally restored (at the anomaly quantum level).

The same conclusion is achieved going to four dimensions, since in this case we get:

$$\begin{aligned}
&\Delta^\natural_{[\mu,\nu,\rho,\sigma]}(x) dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma \\
&= \frac{1}{4!} dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma \frac{\partial}{\partial \mathcal{C}^\mu(x)} \frac{\partial}{\partial \mathcal{C}^\nu(x)} \frac{\partial}{\partial \mathcal{C}^\rho(x)} \frac{\partial}{\partial \mathcal{C}^\sigma(x)} \Bigg(Tr \Bigg(\mathcal{D}\mathcal{C}(x)(\frac{1}{2}\mathcal{C}(x)\mathcal{C}(x)\mathcal{R}(x) + \mathcal{C}(x)\Omega(x) + \mathcal{D}\mathcal{X}(x)) \\
&(\frac{1}{2}\mathcal{C}(x)\mathcal{C}(x)\mathcal{R}(x) + \mathcal{C}(x)\Omega(x) + \mathcal{D}\mathcal{X}(x)) + \frac{1}{2}\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)(\frac{1}{2}\mathcal{C}(x)\mathcal{C}(x)\mathcal{R}(x) + \mathcal{C}(x)\Omega(x) + \mathcal{D}\mathcal{X}(x)) \\
&- \frac{1}{10}\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x)\mathcal{D}\mathcal{C}(x) \Bigg)\Bigg)
\end{aligned} \tag{3.39}$$

and, after performing all the calculations,the anomaly can be rewritten, as well known, in a full covariant way with the aid of total derivatives and coboundary terms,as :

$$\Delta^\natural_{[\mu,\nu,\rho,\sigma]}(x) dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma = \epsilon^{\mu,\nu,\rho,\sigma} \left(-\mathcal{D}_\lambda \mathcal{C}^\eta(x) + \mathcal{K}^\eta_\lambda(x) \right) \mathcal{R}^\tau_{([\mu,\nu],\eta)}(x) \mathcal{R}^\lambda_{([\rho,\sigma],\tau])}(x) d^4x \tag{3.40}$$

and the same result as before in Equation (3.38) is achieved.

We can generalize this result within our approach, since the solution of our problem is revealed noting that

$$\mathcal{S}\left(\mathcal{D}_\nu \mathcal{C}^\mu(x) - \mathcal{K}^\mu_\nu(x) \right) = \left(\mathcal{D}_\nu \mathcal{C}^\lambda(x) - \mathcal{K}^\lambda_\nu(x) \right) \left(\mathcal{D}_\lambda \mathcal{C}^\mu(x) - \mathcal{K}^\mu_\lambda(x) \right) + \overset{\circ}{\Lambda}{}^\mu_\nu(x) \tag{3.41}$$

The previous equation (3.41) is the direct evidence of our results. This rewrites Equation (3.21) putting all the extensions inside the new ghost term $\mathcal{K}^\mu_\nu$, leading to a compensation mechanism which produces the invariants of the non-extended algebra.

This is the reason why in the two and four dimensional anomalies in Equations (3.34) and (3.40) we find the well-known covariant gravitational anomalies [12] [14] with the replacement $\mathcal{D}_\nu \mathcal{C}^\mu(x) \longrightarrow \mathcal{D}_\nu \mathcal{C}^\mu(x) - \mathcal{K}^\mu_\nu(x)$.

With this result, the temptation to adopt a cancellation procedure, is immediate.

As a final investigation, one may ask what is the Classical Topological Action $\Gamma^{Classical}$ invariant under the full extended symmetry; so at the Classical limit we have to verify

$$\begin{aligned}
\mathcal{W}_\mu(x)\Gamma^{Classical} &= 0 \\
\mathcal{W}^\sigma_\mu(x)\Gamma^{Classical} &= 0
\end{aligned} \tag{3.42}$$

This means that the Action is a topological function of the curvature, invariant under usual reparametrizations, and, separately, their extensions in the $\mathcal{K}^{(\mu)}_{(\nu)}(x)$ ghosts.

Now, it is easy to derive from Equations (2.4) , (2.14) and (2.15) that:

$$\mathcal{W}^\tau_\eta(y) \mathcal{R}^\mu_{([\lambda,\rho],\nu)}(x) = \left(\mathcal{R}^\tau_{([\lambda,\rho],\nu)}(x)\delta^\mu_\eta - \mathcal{R}^\mu_{([\lambda,\rho],\eta)}(x)\delta^\tau_\nu \right) \delta(x-y) \tag{3.43}$$

which fix the Action as a function of $\mathcal{R}^{\eta}_{([\mu,\nu],\eta)}(x)$; such as, for example, in $2n$ dimensions the Lagrangian takes the form: $\mathcal{L}^{Classical}(x) = \epsilon^{(\mu_1,\mu_2,\cdots\mu_{2n-1},\mu_{2n})}\mathcal{R}^{\eta_1}_{([\mu_1,\mu_2],\eta_1)}(x)\cdots\mathcal{R}^{\eta_n}_{([\mu_{2n_1},\mu_{2n}],\eta_n)}(x)$.

4 Conclusions

The results we have presented here are very general.Our investigation takes origin only from the curiosity to see beyond a smooth reparametrization, and we make only use of algebraic consistency and stability (B.R.S. nilpotency).The message we derive is that sometimes, just round the corner, we can find the solutions of our troubles.Condition (2.8) is fundamental for this symmetry, not only it shows the right lane which the expansion $\mathcal{X}^{\mu}(x)$ term (from which the anomaly in independent) has to go through, but also reduces the ghosts from infinity to two.

After this shrinking the algebra produces a new symmetry, embodied in condition (3.41), which reconduces the problem to a quiet normality.

Obviously the solution might not to be unique: this gives new forces to the study of the non linear extensions of algebras.

A Appendix

In this Appendix we report the calculations which give the result in (3.28).

Essentially, this comes from the iterations of the formula:

$$\begin{aligned}
&\Lambda^{(r+1)}(x) = \mathcal{S}\Big(\Lambda^{(r)}(x)\mathcal{DC}(x)\Big) - \mathcal{DC}(x)\Lambda^{(r)}(x)\mathcal{DC}(x) \\
&r \geq 0
\end{aligned} \tag{A.44}$$

where we have defined $\Lambda^{(0)}(x) = 1$.

The first iteration, gives:

$$Tr\Big[\Lambda^{(n)}(x)\Big] = \mathcal{S}Tr\Big[\Big(\Lambda^{(n-1)}(x)\mathcal{DC}(x)\Big)\Big] - Tr\Big[\mathcal{DC}(x)\Lambda^{(n-1)}(x)\mathcal{DC}(x)\Big] \tag{A.45}$$

The process goes on as:

$$\begin{aligned}
= &\quad \mathcal{S}Tr\Big[\Big(\Lambda^{(n-1)}(x)\mathcal{DC}(x)\Big)\Big] + Tr\Big[\mathcal{DC}(x)\mathcal{DC}(x)\Lambda^{(n-1)}(x)\Big] \\
= &\quad \mathcal{S}Tr\Big[\Big(\Lambda^{(n-1)}(x)\mathcal{DC}(x)\Big)\Big] + Tr\Big[\mathcal{DC}(x)\mathcal{DC}(x)\Big(\mathcal{S}\Big(\Lambda^{(n-2)}(x)\mathcal{DC}(x)\Big) - \mathcal{DC}(x)\Lambda^{(n-2)}(x)\mathcal{DC}(x)\Big)\Big] \\
= &\quad \mathcal{S}Tr\Big[\Big(\Lambda^{(n-1)}(x)\mathcal{DC}(x)\Big) + \Big((\mathcal{DC}(x)\mathcal{DC}(x))\Lambda^{(n-2)}(x)\mathcal{DC}(x)\Big)\Big] \\
- &\quad Tr\Big[\mathcal{S}(\mathcal{DC}(x)\mathcal{DC}(x))\Lambda^{(n-2)}(x)\mathcal{DC}(x)\Big] - Tr\Big[(\mathcal{DC}(x)\mathcal{DC}(x))\mathcal{D}(x)\Lambda^{(n-2)}(x)\mathcal{DC}(x)\Big] \\
= &\quad \mathcal{S}Tr\Big[\Big(\Lambda^{(n-1)}(x)\mathcal{DC}(x)\Big) + \Big((\mathcal{DC}(x)\mathcal{DC}(x))\Lambda^{(n-2)}(x)\mathcal{DC}(x)\Big)\Big] \\
+ &\quad Tr\Big[\Big(\mathcal{DC}(x)\mathcal{S}(\mathcal{DC}(x)\mathcal{DC}(x)) + (\mathcal{DC}(x)\mathcal{DC}(x))^2\Big)\Lambda^{(n-2)}(x)\Big] \\
= &\quad \mathcal{S}Tr\Big[\Big(\Lambda^{(n-1)}(x)\mathcal{DC}(x)\Big) + \Big((\mathcal{DC}(x)\mathcal{DC}(x))\Lambda^{(n-2)}(x)\mathcal{DC}(x)\Big)\Big] \\
+ &\quad Tr\Big[\Big(\mathcal{DC}(x)\mathcal{S}(\mathcal{DC}(x)\mathcal{DC}(x)) + (\mathcal{DC}(x)\mathcal{DC}(x))^2\Big)\Big(\mathcal{S}\Big(\Lambda^{(n-3)}(x)\mathcal{DC}(x)\Big) - \mathcal{DC}(x)\Lambda^{(n-3)}(x)\mathcal{DC}(x)\Big)\Big] \\
= &\quad \cdots
\end{aligned} \tag{A.46}$$

so we argue a first general result:

$$Tr\left[\Lambda^{(n)}(x)\right] \;=\; Tr\mathcal{S}\left[\sum_{r=0}^{n-1}\Psi^{(r)}(x)\Lambda^{(n-1-r)}(x)\mathcal{DC}(x)\right]+Tr\left[\Psi^{(n)}(x)\right] \tag{A.47}$$

where:

$$\begin{aligned}
\Psi^{(0)}(x) &= 1\\
\Psi^{(1)}(x) &= (\mathcal{DC}(x)\mathcal{DC}(x))\\
\Psi^{(2)}(x) &= \left(\mathcal{DC}(x)\mathcal{S}(\mathcal{DC}(x)\mathcal{DC}(x))+(\mathcal{DC}(x)\mathcal{DC}(x))^2\right)\\
&= \cdots\\
\Psi^{(r)}(x) &= \mathcal{DC}(x)\mathcal{S}\Psi^{(r-1)}(x)+(\mathcal{DC}(x)\mathcal{DC}(x))^{(r)}\\
r &= 1,\cdots n
\end{aligned} \tag{A.48}$$

At this stage we are left here to show that the $Tr\left[\Psi^{(n)}(x)\right]$ is a coboundary, and a new iteration task must be accomplished.

$$\begin{aligned}
Tr\left[\Psi^{(n)}(x)\right] &= Tr\left[\mathcal{DC}(x)\mathcal{S}\Psi^{(n-1)}(x)+(\mathcal{DC}(x)\mathcal{DC}(x))^{(n)}\right]\\
&= Tr\left[\mathcal{DC}(x)\mathcal{S}\Psi^{(n-1)}(x)\right]\\
&= -\mathcal{S}Tr\left[\mathcal{DC}(x)\Psi^{(n-1)}(x)\right]+\left[\left(\mathcal{SDC}(x)\right)\Psi^{(n-1)}(x)\right]\\
&= -\mathcal{S}Tr\left[\mathcal{DC}(x)\Psi^{(n-1)}(x)\right]+Tr\left[\left(\mathcal{SDC}(x)\right)\left(\mathcal{DC}(x)\mathcal{S}\Psi^{(n-2)}(x)+(\mathcal{DC}(x)\mathcal{DC}(x))^{(n-1)}\right]\\
&= \mathcal{S}Tr\left[-\mathcal{DC}(x)\left(\Psi^{(n-1)}(x)+\left(\mathcal{SDC}(x)\right)\Psi^{(n-2)}(x)\right)+\frac{1}{(2n-1)}\left(\mathcal{DC}(x)(\mathcal{DC}(x)\mathcal{DC}(x))^{(n-1)}\right)\right]\\
&+ Tr\left[\left(\mathcal{SDC}(x)\right)^2\Psi^{(n-2)}(x)\right]\\
&= \mathcal{S}Tr\left[\mathcal{DC}(x)\left(\Psi^{(n-1)}(x)+\left(\mathcal{SDC}(x)\right)\Psi^{(n-2)}(x)\right)+\frac{1}{(2n-1)}\left(\mathcal{DC}(x)\left(\mathcal{DC}(x)\mathcal{DC}(x)\right)^{(n-1)}\right)\right]\\
&+ Tr\left[\left(\mathcal{SDC}(x)\right)^2\left(\mathcal{DC}(x)\mathcal{S}\Psi^{(n-3)}(x)+(\mathcal{DC}(x)\mathcal{DC}(x))^{(n-2)}\right)\right]\\
&= \mathcal{S}Tr\left[-\mathcal{DC}(x)\left(\Psi^{(n-1)}(x)+\left(\mathcal{SDC}(x)\right)\Psi^{(n-2)}(x)+\left(\mathcal{SDC}(x)\right)^2\Psi^{(n-3)}(x)\right)\right.\\
&+ \left.\frac{1}{(2n-1)}\left(\mathcal{DC}(x)\left(\mathcal{DC}(x)\mathcal{DC}(x)\right)^{(n-1)}\right)+(\mathcal{SDC}(x))\frac{1}{(2n-3)}\left(\mathcal{DC}(x)\left(\mathcal{DC}(x)\mathcal{DC}(x)\right)^{(n-2)}\right)\right]\\
&+ Tr\left[\left(\mathcal{SDC}(x)\right)^2\left(\mathcal{DC}(x)\Psi^{(n-3)}(x)\right)\right]\\
&= \cdots\\
&= \sum_{r=0}^{(n-1)}\mathcal{S}Tr\left[\mathcal{DC}(x)\left(-\left(\mathcal{SDC}(x)\right)^{(n-1-r)}\Psi^{(r)}(x)\right)\right)\\
&+ \left.\left(\mathcal{SDC}(x)\right)^{(n-1-r)}\frac{1}{(2r+1)}\left(\left(\mathcal{DC}(x)\mathcal{DC}(x)\right)^{(r)}\right)\right]
\end{aligned}$$

$$ \tag{A.49} $$

where we have used:

$$TrS\left[\left((\mathcal{DC}(x)\mathcal{DC}(x))^{(n-r)}\right)\mathcal{DC}(x)\right] = (2(n-r)+1)Tr\left[\left(\mathcal{S}\mathcal{DC}(x)\right)\left(\mathcal{DC}(x)\mathcal{DC}(x)\right)^{(n-r)}\right] \tag{A.50}$$

and the iterations ends with:

$$Tr\left[(\mathcal{DC}(x)\mathcal{DC}(x))^{(n)}\right] = 0 \tag{A.51}$$

so our final result in Equation (3.28) is obtained.

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